SMALL GROUP LEARNING IN MATHEMATICS

Teachers' and pupils' ideas about groupwork in school

PASI SAHLBERG & JOHN BERRY

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Foreword

Small group learning promotes learning. This statement seems redundant. In the context of a review of research on teaching mathematics it intends to express the observation that schools often stress teaching at the expense of learning. Creating the conditions for *learning* and not just for *teaching* should be the highest priority of schools. In today's educational world that is *not* the case.

Co-operative learning suggests a variety of methods for managing and promoting student learning by enhancing mutual assistance among students so they will work together to solve problems and clarify ideas. Co-operative learning also seeks to stimulate students' motivation to learn by having them assume responsibility for making many of the decisions involved in the process of learning, and not reserve that responsibility exclusively for the teacher.

Extensive research, much of it in the form of classroom-based experiments, assessed whether these approaches to teaching and learning fulfil their goals. By and large the results are very encouraging, demonstrating the significantly superior effects on student learning of co-operative learning methods compared, first and foremost, to the presentation-recitation form of instruction that dominates the majority of classrooms throughout the world (Slavin 1995).

Readers of this book should recall that implementing a new teaching method in ongoing classrooms, where teachers had little or no previous experience with the new methods and learned them recently in an in-service training experience, is a singularly hazardous and difficult task. Its success is decidedly uncertain. Moreover, a classroom is only one unit in a large institution called a school, and is subject to a myriad of constraints impinging upon it from the school's regulations and organization that present no small impediment to instructional innovation (Sarason 1990; 2002; Sharan et al. 1999). Given this complex condition of classroom teaching, the fact that classroom-based experiments on co-operative learning, carried out for relatively limited periods of time on distinct disciplinary topics, yielded such obviously positive findings about their effects on student learning and attitudes, is testimony to its powerful impact. Co-operative learning, in its various manifestations, introduces significant *changes in the behaviour* of everyone involved in classroom learning. Those changes bring in their wake measurable changes in student learning. Students' academic achievement in any subject will not change or improve noticeably if classroom learning conditions remain substantially unchanged. Despite this seemingly obvious truism, school systems insist on assessing the end product called achievement, and largely disregard the nature of the process known as learning, *as if achievement is independent of learning!*

The goal of co-operative learning to promote students' learning requires renewed concentration of educators on the learning process as it can be carried out in the environment known as the school and the classroom. That environment is above all a social one that encompasses many people of different age levels who must interact with one another. It is also, as this book emphasizes, necessary to rethink the task and assignments in which small groups are engaged. Co-operative learning recommends taking advantage of the social dimension of schooling and school learning to enhance and motivate students.

The emphasis on the social nature of the school environment should convey the message to educators that the specific contents of the academic discipline they may wish to teach does not determine the nature of the social environment in which teaching is to occur, and hence it does not exert a determining influence on how learning will take place. If educators will focus their major professional efforts on the content of their discipline, and continue to ignore the nature of the social environment in which students experience the process of learning, education will continue to miss the boat and successfully bore the life out of our youth (Sarason 1983; 1990).

No one advocates the notion that teachers can be ignorant of their academic discipline and still teach well. No one argues against the idea that superior knowledge and perspective on mathematics is relevant to the teaching of mathematics. Obviously, if the contents of textbooks or other resources used by students are muddled, badly presented and explained, and so forth, few will make sense out of it. That is true for all "learners" at all ages in all walks of life. The claim made here is that, in and of itself, knowledge of a discipline cannot improve learning unless teachers can significantly improve the social-psychological dimensions of the learning environment and of the process of learning. That knowledge is not to be found in the substance of the traditional academic disciplines per se, such as mathematics, chemistry, history and so forth. If that were the case, then co-operative learning methods, or any other genuine innovation in the *method of teaching and learning,* would not make any difference. Yet, what research of the kind reviewed in this book tells us is that the instructional method *does* make a difference for student learning. That difference can stem only from the implementation of an innovative method of teaching and learning, and not from changes in academic content alone, no matter how well it is worked out by educators in advance of teaching and learning. Indeed, Sarason's (1996) insightful presentation of how several decades ago "the new math" failed to produce any of its desired results confirms that conclusion unequivocally.

It is quite clear that universities will not relish the implications of the conclusion reached here about the role of academic disciplines in the shaping of instruction. The preparation of teachers in institutions of higher learning is more often than not focused on the various disciplines as unrelated and separate domains. Disciplinary experts rarely consider these domains to be intimately related to basic aspects of school learning, motivation to learn, classroom organization for learning, student cooperation to learn, to other disciplines, and a host of other features of life in schools and classrooms that directly or indirectly affect learning (Sarason 1993). Certainly these and related topics are not considered as related to the specialized domain of university personnel associated with a given discipline "above and beyond" the methods of instruction. Teachers' expertise, particularly in secondary schools, is defined by their disciplines, not by their role as teachers of people. The students' role is to learn, i.e. "absorb" the disciplinary contents allocated by the authorities to given age or grade levels.

Some educators may object that this is an extreme formulation and that teachers in schools with practical experience do not share that perspective. Perhaps that is so to some extent. Nevertheless, many, if not most, universities prepare their teachers in precisely that fashion, and a very large percentage of practicing secondary school teachers perform their professional task in a manner highly consistent with the description offered here, whether or not they would agree with the verbal formulation. If the conclusions of this book are taken seriously, most of the teacher educators in universities need to think again not only about their curricula but also about the methods used in preparing teachers to teach mathematics in an inspiring way.

Co-operative learning emerges from the discussion thus far as a radical departure from the traditional instructional scenario. Perhaps that is why many school systems and teachers talk a lot about co-operative learning but implement it rarely or the implementation is so shabby that it would be best ignored. A recent survey of 142 mathematics teachers in Finland revealed that more than 80% regularly employed traditional teacher-centred, text-book centred, lecture-centred instructional methods, while less than 25% said they employed “active learning” that engaged students in group-assisted problem solving. The challenge of restructuring the classroom, the teacher’s professional behaviour, students’ expectations, and so forth, were highlighted in the Finnish report (Röj-Lindberg 2001). Even more remarkable findings about the absence of change were published in a national survey of high school principals in the United States regarding the implementation of alternative instructional strategies and of restructuring procedures (Cawelti 1994). Maybe it is no surprise then that the research report in hand comes to similar conclusions. As noted, teachers must reorient themselves to think about instruction in mathematics as a form of human experience, and not as a collection of abstract symbols to be mentally manipulated by techniques that students have thoroughly memorized.

Similar statements about the need for reorienting teachers’ views of their discipline with respect to its impact on teaching and learning could be made about the study of all the sciences or technologies taught in schools. Instruction in the Humanities seems less afflicted with the malaise of disciplinary isolationism. However, that does not translate directly into teachers’ adopting group-centred co-operative learning or any other version of student active learning in the classroom even though the teachers' may be conceptually prepared to accept the alternative approach. For example, second-language instruction has many built-in rigidities that impede the adoption of co-operative or active learning, despite the emphasis in the theory of second-language learning on the need for communication as the preferred vehicle for learning a new language. Second language instruction, the teaching of mathematics, and instruction in other disciplines as well, include the conviction that teachers must instruct (i.e. tell) students first in the proper, error-free, skills and information. Until this foundation of basic facts is set in place, students cannot be left to their own devices to interact with one another, raise their own questions, explore the ramifications of their own curiosity, or deal with tasks that require the use of various skills deriving from the given discipline. Those skills must be acquired *first*. Only then students can proceed to deal with "interesting" problems. That is the academically correct sequence of instructional conduct.

The distinctly beneficial effects of co-operative learning in the cognitive and affective domains can be reaped in far greater scope and depth than currently reported in research. Again, such benefits accrue when major elements in the educational environment have undergone changes that enhance the adoption and implementation of co-operative learning. These topics can be discussed here in telegraphic fashion only. Readers must consult the relevant publications for a thorough presentation.

The basic changes needed to make a quantum leap in contemporary teaching and learning relate to:

1. the nature of the curriculum, its scope, the tasks and basic conception of learning. Related disciplines can be combined in a variety of ways to produce a trans-disciplinary, problem-centred approach to teaching and learning, instead of the current single-disciplinary approach.

2. aspects of staff organization. Teachers can work together in multi-disciplinary teams, to plan and implement instruction, and facilitate a co-operative, group-centred, problem-centred, intellectually broad and personally meaningful approach to learning for students.

These two elements link classroom learning to its broader school-wide context. Those elements affect the design of the learning environment so that school-classroom-curriculum-students are coherently coordinated and can function with a high level of effectiveness and satisfaction (Sharan et al. 1999).

Shlomo Sharan
Netanya, ISRAEL
November, 2002

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Introduction

Johnny likes mathematics but he doesn't like the mathematics lessons in his school. The mathematics teacher often has to ask Johnny to focus on work rather than talking to the other boys sitting around him. All the pupils in Johnny's class are sitting in groups of four and this is the permanent arrangement in his mathematics class. According to the teacher the purpose of this seating arrangement is to promote co-operative learning and groupwork in mathematics in order to make mathematics easier and more enjoyable for the pupils. This is a 6^{th} grade class (pupils aged 12) having 25 boys and girls full of life, anxiety and high expectations.

Johnny is always sitting with the same boys. Their group has moved to the back of the classroom, a distance from the teacher. It is the boys' own choice where and with whom they sit. They decided to form that group at the beginning of the school year. They know each other well and are actually best friends out of school. Their group is a safe place for them among the other pupils and it also encourages them to express themselves more freely. But the teacher is not happy with how the boys are behaving in general and succeeding in mathematics in particular.

This reflection from real life seems to be quite common in our schools. Pupils have been seated in small groups but they still study and learn individually. This notion of inappropriate use of small groups in mathematics is one of the points of departure for this study that we present in this book. We have met hundreds of teachers who claim that co-operative learning is not appropriate in teaching and learning mathematics. Their evidence comes from their own classrooms. The teachers often report that small groups in mathematics lessons encourage non-productive behaviour, decreasing time on task, frustration among higher-ability pupils, and most of all general uncertainty concerning learning and achievement. Teachers who are teaching mathematics are generally interested in learning more about co-operative learning because basically none of them have received any training on the methods during their teacher preparation in colleges and universities. The main challenge for teachers has been the lack of suitable subject-specific materials and resources to use in small group learning situations.

This research reported here began some six years ago when we began to merge mathematical problem solving and co-operative learning in teacher in-service training workshops. We thought that solving problems and doing mathematical investigations provided a natural context for pupils not only to sit and work together, but also to learn mathematics together with other pupils. The teachers with whom we were working often tried to convince us that co-operative learning is already implemented in their schools and is also widely practiced in mathematics lessons. We learnt that as a first step to transform their mathematics classrooms into co-operative learning classrooms, teachers assigned pupils to sit together, often in groups of four. Teaching mathematics from the front of the classroom and pupils studying on their own became almost a sin among educational reformists. However, we only very rarely met a mathematics teacher who was satisfied with that arrangement. Thus, the belief in the power of co-operation and learning mathematics together started to vanish. And this is no surprise.

After six years of intensive work with teachers teaching mathematics in schools in England and Finland we think that something is fundamentally wrong in thinking about co-operative learning in mathematics. We have become cynical with the simple solutions that have been proposed to improve the teaching and learning of mathematics. This seems to be particularly true in making learning mathematics more interactive and socially active for our pupils. The inappropriate use of small groups may actually do more harm than bring improvements. Pupils very quickly learn the rules of the game – they know what small groups are there for. Many pupils think that co-operative learning is ineffective or useless in terms of productive learning in school. Changing pupils' beliefs and establishing new rules are always difficult and demanding processes. Therefore, we suggest that traditional seating order and classroom arrangements would be used whenever appropriate techniques and methods of co-operative learning or working together are not utilised.

We use in this book the words 'co-operative learning' and 'small group learning' interchangeably. Co-operative learning often refers to specific methods of teaching or schools of thought among the umbrella of groupwork. 'Small group learning' that we use in the title of this book is, we believe, more generic and free from any previous traditions or techniques. As we have described earlier, co-operative learning has gained the burden of an 'already implemented' innovation in education that does not deserve further attention. Small group learning in this book simply means dividing the whole class into teams of two to four pupils who will be assigned specifically designed interactive tasks.

We would like to thank all teachers with whom we have had many wonderful opportunities to co-operate in a number of workshops, seminars and meetings. We have learnt a lot from them and it has helped us to create a clearer picture of the learning of mathematics in our schools. Moreover, our gratitude goes to those hundreds of pupils who shared their ideas with us and opened the windows to the fascinating world of learners. Finally, we are pleased to express our thanks to our colleagues Jenny Sharp and Heta Tuominen for interviewing the teachers.

Chapter 1

The teaching and learning of mathematics

In many countries, teachers of mathematics are being encouraged to move from presentation-recitation modes of teaching towards a blend of instructional methods. An increasing emphasis on real problem-solving, investigations, projects and other forms of applying mathematical knowledge and skills in everyday life situations is changing the nature of mathematics in schools. There has been a growing demand from both professional and business people and education policy-makers to stimulate active learning, promote effective teaching, and encourage appropriate assessment methods to be utilised in the teaching and learning of mathematics (Black & Atkin 1996). Active learning has been interpreted to mean more participatory learning methods, such as communication, co-operation and working on real-life problems. Therefore, many teachers and school improvement experts seeking for better quality teaching and learning have turned their eyes to small group learning methods that have been developed since the late 1960s (Sutton 1992; Robertson et al. 1994; Webb & Farivar 1994; Owens 1995; Ogden 2000).

Alternative elements to the traditional teaching of mathematics have created pressure to break the predominant presentation-recitation model of teaching mathematics in schools. Since the 1980's problem solving has been a widely accepted and implemented feature of change in traditional teaching and learning of school mathematics. During this time problem solving has become an international field of research in mathematics education (Schoenfeld 1985; Mason et al.

1982; Mason 1991). Moreover, the emergence of new learning tools, such as computers and hand-held technologies have enriched the traditionally one-way communication structure of classrooms towards multi-lateral interactive learning environments in which new abilities and attitudes are necessary in order to create productive settings for learning and understanding. Another area of activity of mathematics education research is therefore information and communication technologies (ICT) in mathematics. However, our study reported here focuses on small group learning in mathematics that is becoming another important area of research as a response to the ongoing demands in raising the quality of teaching and learning, and as a consequence pupil achievement in schools worldwide.

Restructuring whole classes into small groups of pupils has been seen as one pedagogical response to these challenges. In this research report we use the term co-operative learning to refer to dividing a large group into groups of two to six pupils and assigning specific tasks to these groups (Sharan 1999). However, this kind of general definition may be confusing because there are in fact several different models of co-operative learning that vary considerably by their epistemological orientations and practical implications for the roles of the teacher and the learners. Particularly in mathematics, cooperative work can be used in conjunction with practising skills, doing investigations, collecting data, discussing concepts and principles, or solving mathematical problems (Davidson & Kroll 1991; Cobb 1995).

1.1 CURRICULUM REFORMS IN FINLAND AND ENGLAND

Although the reforms of teaching school mathematics have similarities in terms of global trends, their implications on classroom practice may be different from one place to another. In Finland, for example, the National Framework Curriculum for Comprehensive School (Anon. 1994) placed a strong emphasis on the constructivist approach to teaching mathematics and on the utilisation of interactive-intensive learning arrangements, such as discussions and problem solving in real-life contexts, in schools. However, teachers in schools have been struggling to find appropriate instructional methods to respond to these new pedagogical and professional requirements. As there is no reliable evidence of the prevalence of co-operative learning among mathematics teachers today, one of the purposes of this study is to find out how mathematics teachers use small group learning in their lessons and what are the reasons for doing so. Our assumption is that despite an increased emphasis on problem solving and practical mathematics in Finnish schools, the methods of co-operative learning in mathematics are not well advanced.

Recent developments in England are also interesting. Until recently there has been a fairly broad range of teaching and learning styles in English schools with group work and investigations encouraged at all levels and particularly at primary school level where mathematics was often integrated as part of the curriculum with project work featuring strongly. However recent developments have changed the freedom of teachers to teach in a style conducive to good learning. The emphasis on testing and the introduction of the National Numeracy Strategy (1999) in primary schools in the year 2000 and in lower secondary schools in September 2001 has meant that not only is the curriculum prescribed but the teaching style is also prescribed by central government.

The immediate aim of the National Numeracy Strategy (NNS) was to “raise standards in mathematics” in primary schools as measured by standard tests for 11 year olds. This was very much a political agenda in response to a perceived need to raise the level of mathematical attainment of primary children. Various things have driven this apparent need to “raise educational standards”, in particular the international comparisons between England and other countries. The Third International Mathematics and Science Survey (TIMMS) apparently highlighted weaknesses in the mathematics in English schools especially in the area of number.

A feature of the National Numeracy Strategy is the drive for ‘whole class teaching’ and away from pupils working alone from books or worksheets and away from ‘project work’ as a means of delivering the curriculum. Whole class teaching, although being one of the important ideas to emerge from the international comparisons, is more than the teacher teaching and pupil watching. Tanner and Jones (2000) provide a good summary of the features that should be included in a ‘typical’ daily mathematics lesson that schools should have as part of the NNS. It is too early to say how successful the NNS will be. With the emphasis of 'teaching to the tests' it is inevitable that test scores will improve and the government will say that the NNS has worked however will pupils develop an understanding of the basic skills and a view of mathematics which is anything other than a rule based subject useful in tests?

1.2 MATHEMATICS AND MATHEMATICIANS THROUGH PUPILS' EYES

Often there is a tension between the school curriculum and the perceived needs of college and university mathematicians. Too often the mathematics curriculum at all levels is seen by teachers and pupils as a 'body of knowledge' which needs to be delivered in order to provide an 'acceptable graduate in mathematics'. When asked the question "What is Mathematics?" the most common answers from pupils include: numbers, algebra, a way of thinking, a tool for solving problems.

Mathematics is a subject that is often held in low esteem by children and adults, as Lim and Ernest (1998) summarize:

> It is a matter of concern that ... negative images of mathematics might be one of the factors that has led to the decrease in student enrolment in mathematics and science at institutions of higher education, in the past decade or two ... the term 'image of mathematics' refers to a mental picture, view or attitude towards mathematics, presumably developed as a result of social experiences, through school, parents, peers, mass media or other influences.

Pupils and their teachers are as affected by society and the media's views of mathematics as anyone, and the image generally portrayed of mathematics and mathematicians is not a good one. Battista (1999) reported an often-quoted example of explaining to someone that one teaches mathematics seldom elicits any other response than,

> 'I was never good at math,' as if displaying a badge of courage for enduring what for them was a painful and useless experience. In contrast, people do not freely admit that they can't read.

As part of a research project at the University of Plymouth, we have been studying students' perceptions of mathematicians and what they do. We have asked groups of students from the USA and Europe to draw a mathematician and to think of reasons for hiring a mathematician (see Picker and Berry 2000). Figure 1.1 is a typical example of the student drawings.

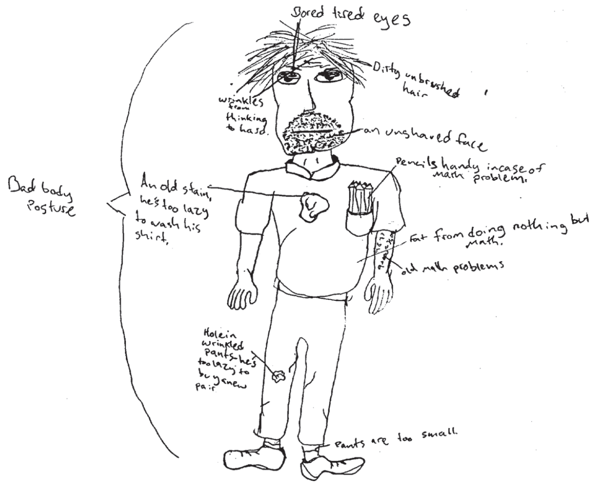

FIGURE 1.1 A COMMON CHILDREN'S IMAGE OF A MATHEMATICIAN

As well as drawing a picture we asked the pupils to write some words to describe their image of a mathematician. The pupil who drew figure 1.1 wrote:

"Mathematicians

- have no friends (except other mathematicians)
- are not married or seeing anyone
- are usually fat
- are very unstylish
- have wrinkles in forehead from thinking so hard
- have no social life whatsoever

- are 30 years old
- have a very short temper"

These images are fairly common from the countries in our study; England, Finland, Germany, Romania, Sweden, USA and Wales. It is important to ask: Should we be concerned about these images? What do they say about the pupils who have them and their motivation for learning mathematics and becoming mathematicians? Where do these images come from? What do they tell us about pupils' knowledge about what mathematicians really do? These questions are explored in some detail in the research thesis (Picker 2000).

Figures 1.2–1.6 show further examples of pictures drawn by pupils and taken from the research work.

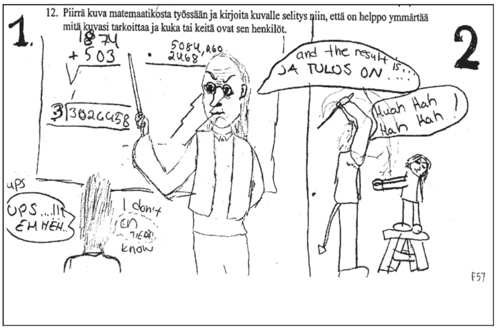

FIGURE 1.2
FINLAND—FEMALE PUPIL

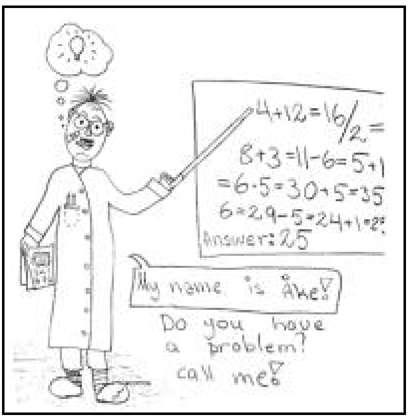

FIGURE 1.3
SWEDEN—FEMALE PUPIL
(THE FOOLISH MATHEMATICIAN)

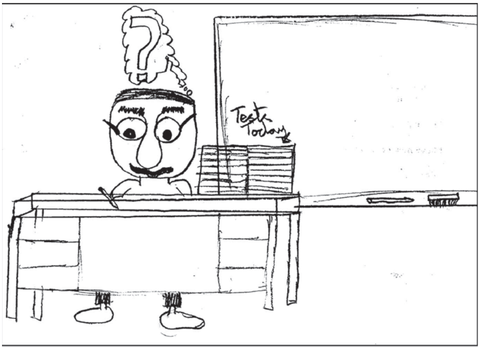

FIGURE 1.4
UK—MALE PUPIL (SHOWING THE UK'S PREOCCUPATION WITH TESTING!)

FIGURE 1.5
U.S. – MALE PUPIL (THE MATHEMATICIAN WHO CANNOT TEACH)

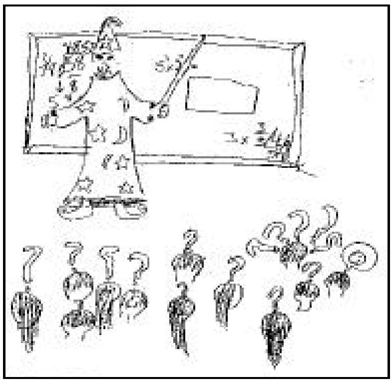

FIGURE 1.6
THE MATHEMATICIAN WITH MAGIC POWERS

At the same time we asked the pupils why one might need to hire a mathematician. The responses to this request were very skimpy. Pupils mentioned teaching and tutoring, doing taxes, working in a bank or shop, "to solve hard problems" (USA pupils) "to do hard sums" (UK pupils) although no specific type of "hard problem" or "hard sum" was ever mentioned. By and large it does not seem to be at all clear to pupils what it is that mathematicians do and what types of problems they can solve. Malkevitch (1997) states that

> the bottom line for many students is that despite being exposed to mathematics continuously from Kindergarten...the typical [student] cannot connect the value of the study of mathematics with what mathematicians really do. Put differently, students have learned when to "call" or hire a doctor, electrician, geologist, or plumber, but not when to "call" or hire a mathematician

Those of us who enjoy mathematics, and enjoy teaching it, would probably like to challenge and change the negative views portrayed in such images. Perhaps we need to bring into the classroom more examples of mathematics in use and mathematicians at work.

The largest finding of our study is that for pupils of this age, mathematicians and what they do are essentially invisible, with the result that, when asked, pupils appear to rely on stereotypical images from the media to provide images of mathematicians. Pupils believe that mathematicians do applications similar to those they have seen in their own mathematics classes, including arithmetic computation, area and perimeter, and measurement. They also believe that a mathematician's work involves accounting, doing taxes and bills, and banking; work which they contend includes doing hard sums or hard problems; yet pupils can supply no specifics about what such problems entail. Furthermore all of the images showed mathematicians working alone in isolation.

A number of the Romanian students expressed this idea of a mathematician as a sort of sorcerer's apprentice, taking exams, doing homework, teaching "math tricks," as a thirteen-year old student expressed it. This notion of a mathematician as a magician is a common theme and one that we ought to reflect upon.

In providing the images with our survey tool, we could not have anticipated how much pupils would provide a window onto their experiences in their mathematics classes. We believe that the drawings created by the pupils contain valuable insights with significant implications for teachers, their training and their practice.

One of the most surprising and startling images pupils drew in almost every country is one of small children powerless before mathematicians who were drawn as authoritarian and threatening. Pupils appeared to use experiences of having been intimidated in mathematics classes and their criticisms of teachers for doing this, at times to depict mathematicians in their drawings in a vengeful manner, something with which they were aided by images of mathematicians in the media.

Teachers appear to be largely unaware of their pupils' lack of knowledge about mathematicians and the role they can play both in shaping and in changing their pupils' views about them. Solving real problems and developing mathematical thinking through investigations in the school curriculum can be one intervention strategy to challenge pupils' views of mathematicians and what they do. Moreover, early experiences of co-operative doing and learning of mathematics may have long-term effects on pupils' images of mathematics and their decisions concerning further studies.

1.3 Dilemma of teaching mathematics

This study originates from a joint interest in developing the teaching of mathematics in English and Finnish schools. One of us had walked a long path by searching and then researching problem solving, modelling and investigations in school mathematics without and with modern technological tools. The other one had developed and researched alternative approaches to teaching and learning in general educational contexts through various teaching and learning methods, especially co-operative learning. These paths crossed in the late 1980s and by the middle of the 1990s we had integrated our conception of mathematics teaching and learning as a dynamic, interactive-intensive school subject. The driving force behind our culturally and also scientifically mixed intention was to understand the nature of the dilemma of teaching mathematics: how do we increase open-ended, co-operative working styles in mathematics lessons in our schools while the traditions and commonly held beliefs about mathematics support isolated thinking and working rather than socially constructed understanding and collective reasoning? Furthermore, the education policies in England and Finland have created very different frameworks for action in terms of how teaching is expected to be arranged. Whereas the Finnish education policy and national curriculum guidelines encourage teachers to seek alternatives to traditional teacher-centred pedagogies, in England the increased accountability and external testing of pupils have narrowed the options of teachers to choose appropriate methodologies for teaching. This has also led to overcrowded curriculum specifications allowing teachers little time to do anything interesting.

Mathematics is often seen as a subject to be learnt and worked at individually and in isolation from other people. For example, recent research on student-teachers views indicates mechanistic and static conceptions in terms of the nature of mathematics (Pietilä 2002). Teachers' own experiences as students in university and school have established the roots for these beliefs. The dilemma of teaching mathematics is about the tension between teachers' previous experiences and related conceptions of mathematics, and expectations expressed by the curriculum, employers and pupils themselves of more dynamic working and learning styles in mathematics.

A major challenge to the introduction of small group learning in mathematics classes is that those entering the teaching profession often do not have the mathematical or pedagogical knowledge necessary to teach in an investigative way. Because of the common view of mathematics as a body of knowledge (number, algebra and geometry) school leavers have learned some content, often using traditional teacher-centred learning methods, that can be carried out more efficiently by information communication technology (ICT). Our own experience during the past ten years of delivering inservice training courses for elementary, middle and secondary school teachers has shown that few teachers have acquired the skills of conjecture, exploration and enquiry that are important for developing good mathematical thinking. When given a complex problem the notion of simplify, generalise and verify does not come naturally. We would argue that most teachers when teaching mathematics will 'do unto them as was done to them' when they were pupils at school. Klein (2001) confirms this view when investigating constructivist practice in mathematics classrooms.

Another factor that needs to be noted here is the nature of teachers' general beliefs of teaching and learning. According to Prawat (1992) teachers' ideas of teaching and learning are often fixed and based on a mechanistic epistemological orientation (also Sahlberg 1996). These pedagogical orientations together with rather narrow conceptions of the subject itself create a frame of reference that supports the view of teaching and learning mathematics as one-way communication, mastering knowledge and procedures, and being able to cope with mathematical problems independently. If these conditions exist, it is difficult to convince mathematics teachers that pupils learn mathematics, i.e. mathematical concepts, procedures and skills more effectively in small groups if certain critical elements are included in groupwork situations. Instead, teachers may choose to use small groups as an alternative to typical teacher-led teaching and provide their pupils with an opportunity to work together to use mathematics for example in standard exercises or problem solving tasks. In other words, it appears to be easier to persuade teachers to use small groups to do mathematics but not to learn new mathematical knowledge or skills.

The purpose of the present study was to investigate to what extent and why small group learning is used in mathematics and to draw more educators' and researchers' attention to the importance of task design and reformulation in small group learning in school mathematics. Based on informal discussions with several hundred teachers of mathematics we have established a view that small group learning is not used in ways that promote high quality interaction in mathematics lessons. The anecdotal evidence from our discussions is that pupils may often be sitting in groups (most commonly pairs but sometimes in triads) but are mostly working at mathematics alone. We also have little knowledge about the tasks that are used in the previous research studies on co-operative learning and mathematics. Regrettably, most publications describing research on small group learning or co-operative learning do not specify the tasks that are used in the research studies. We would argue that the tasks are an integral part of the learning of mathematics and where experiments involving styles of teaching and learning are carried out then the task design is an important variable. On what grounds can research compare the effectiveness of co-operative learning and traditional forms of learning when the tasks are essentially those used for more individual styles of learning? The omission of a description of the role of the task and task redesign for co-operative learning methods makes it very difficult to judge the outcomes of research on co-operative learning and mathematics.

In this study we wanted to examine the role of small group learning in school mathematics. One of the difficulties in approaching this issue is a conceptual one: What is small group learning? In the literature co-operative learning has become widely known and a commonly used pedagogical approach worldwide. It is sometimes incorrectly linked to some particular single small group learning method, such as the Jigsaw. Since there are no agreed meanings for co-operative learning, groupwork or collaborative learning, in this monograph we use the term small group learning to mean any pedagogical arrangement that divides a large group into smaller units of, for instance 2 to 6, pupils and utilises certain principles and techniques to promote learning by the group members.

In chapter 2 we trace the factors that contribute to the popularity of co-operative learning in general and discuss the conceptual terminology of small group learning. Chapter 3 reviews the research literature on small group learning in school mathematics. Our purpose in this review is to bring together key research findings from various perspectives and to identify the experience of small group learning in school mathematics and the types of tasks used in previous research. The role of the tasks in teaching and learning mathematics at all levels has important influences on the nature and view of mathematics held by the teacher and learner. In chapter 4 we propose a classification of mathematics tasks linked to the good ingredients of teaching and learning mathematics and relate the concept of equal exchange models (Cohen 1994) to this typology of tasks. We then describe the design for the research, data collection and report the results of the census questionnaires of teachers and pupils and interviews of teachers from both Finland and England. In chapter 7 we propose three outcomes from the research study and we conclude with a discussion of the implications of the research for the use of small group learning in teaching and learning with proposals for further research.